

# NEUTRINOS AND STRUCTURE OF THE INTERMEDIATE MASS SCALE

Alexei Yu. Smirnov

*Yukawa Institute for Theoretical Physics*

*Kyoto University, 606-Kyoto, Japan*

*International Centre for Theoretical Physics*

*P. O. Box 586, 34100 Trieste, Italy*

*Institute for Nuclear Research, Russian Academy of Sciences*

*117312 Moscow, Russia*

## Abstract

Neutrino data lead via the see-saw mechanism to masses of the right handed neutrinos at the intermediate mass scale. Simple formalism is suggested which incorporates the quark - lepton symmetry and allows one to find properties of the intermediate scale (masses and mixing) from neutrino data. Averaged mass scale and the mass hierarchy parameter are introduced and fixed by the data. They determine natural ranges of masses and mixing at the intermediate scale. In particular, scenario which includes the MSW solution of the solar neutrino problem and tau neutrino as the hot component of Dark Matter of the Universe leads to  $M_2 = (2 - 4) \cdot 10^{10}$  GeV and  $M_3 = (4 - 8) \cdot 10^{12}$  GeV in agreement with the linear mass hierarchy. Strong deviations from the natural ranges imply fine tuning of parameters or/and certain symmetry of the Majorana mass matrix of the right handed neutrinos.

# 1 Introduction

Neutrino data indicate an existence of the intermediate mass scale  $M \sim 10^{10} - 10^{13}$  GeV. This statement is based on the following assumptions:

1. Neutrino masses are generated by the see-saw mechanism [1]. Mass matrix of light Majorana neutrinos,  $m_\nu$ , has the following form

$$m_\nu \approx -m_D M_R^{-1} m_D^T. \quad (1)$$

Here  $m_D$  is the neutrino Dirac mass matrix and  $M_R$  is the Majorana mass matrix of the right handed (RH) neutrino components <sup>1</sup>.

2. There is a quark-lepton symmetry (or analogy), according to which the Dirac mass matrices of leptons are similar to mass matrices of quarks. In particular, the Dirac neutrino matrix,  $m_D$ , is similar to mass matrix of the up quarks:  $m_D \sim m^{up}$ . This results in equality or certain relations between quark and lepton masses. The relations are generic consequences of Grand Unification, but they also often appear in string inspired models.

3. Deficit of solar neutrinos is due to the resonance flavor conversion (MSW)  $\nu_e \rightarrow \nu_\mu$ . Large scale structure of the Universe is explained in terms of Cold plus Hot Dark Matter (HDM) scenario which implies at least one (presumably, third) neutrino mass in the range 2 - 10 eV. The atmospheric neutrino problem has a solution in terms of oscillations  $\nu_\mu - \nu_\tau$ . Each of these three hints separately leads via the see-saw relation,  $M \sim m_D^2/m$ , to the Majorana masses of the RH neutrinos in the intermediate scale.

What is the origin of the intermediate scale? Do the masses of the RH neutrinos related to masses from other sectors of theory at the intermediate scale?

There is a number of proposals: (i) Intermediate scale can be the intermediate gauge scale: e.g. the scale of L - R symmetry violation [2]. (ii) It could be related to the Peccei-Quinn symmetry breaking scale [3]. (iii) It was marked that RH neutrino masses are in the range of the supersymmetry breaking scale in the hidden sector. (iv) The intermediate scale can originate from GUT scale and higher mass scales, e.g. Planck scale, as  $M_I \sim M_{GU}^2/M_{Pl}$ . (v) On the

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<sup>1</sup>In models with intermediate scale direct Majorana masses of left components are typically much smaller than those from (1).

contrary, it could be constructed from much smaller scales, when more than two states from each generation participate in the see-saw.  $M_I \sim M_D^2/\mu$  with  $\mu \sim m_{3/2}$  and  $M_D \sim 10^7$  GeV. (vi) The scale can appear as radiative correction to the GUT scale in the non-susy GUT [4]. (vii) It could be just due to smallness of the corresponding Yukawa couplings, although such a possibility seems quite unnatural, since even for third generation the coupling should be of order  $10^{-3}$ .

The existence of the intermediate scale is crucial for possible unification of particles and interactions. And to identify the origin of the scale one needs to know a detailed information on its structure (masses, mixing). In the most of studies  $M_R$  is fixed by some ansatz or its structure is related by symmetry to the structure of quark mass matrices. This allows one using the see-saw mechanism to make predictions of masses of the light neutrinos. In this paper we consider the opposite task: determination of masses and mixing of the right handed neutrinos from the low energy neutrino data. In the specific context of the SO(10) model the problem has been solved in [5]. In contrast we will study properties of the intermediate scale within the general model independent framework (1 - 3).

The paper is organized as follows. In sect.2 the see-saw mass matrix is defined and main points of the approach are formulated. In sect. 3 we get a relation between the eigenvalues of mass matrices entering the see-saw formula. In sect.4 mass parameters in absence of mixing at the intermediate scale are determined. These parameters in turn determine the average scale and mass hierarchy. The possibility of the universal mass scale is considered in sect.5. In sect.6 we study the influence of mixing at the intermediate scale on mass hierarchy and on mixing of the light neutrinos. The task is solved for two (second and third) generations. The effect of first generation is estimated in sect. 7. Sect. 8 summarizes the results.

## 2 The see-saw mass matrix

Let us factorize the lepton mixing matrix in the following way [6]

$$V_l = V_D \cdot V_s , \tag{2}$$

where

$$V_D \equiv S_l^+ \cdot S_\nu , \tag{3}$$

and  $S_l$ ,  $S_\nu$  are the transformations which diagonalize the Dirac mass matrices of charge leptons and neutrinos correspondingly. The matrix  $V_D$  is a direct analogy of the CKM-matrix of quark mixing, whereas  $V_s$  specifies the effect of the see-saw mechanism, i.e. the effects of the Majorana mass matrix of the RH neutrino components. As follows from (1)  $V_s$  is determined by diagonalization

$$V_s m_{ss} V_s^+ = \text{diag}(m_1, m_2, m_3) \quad (4)$$

of matrix [6]

$$m_{ss} = -m_D^{\text{diag}} M_R^{-1} m_D^{\text{diag}}. \quad (5)$$

Here  $m_i$  ( $i = 1, 2, 3$ ) are the masses of light neutrinos and

$$m_D^{\text{diag}} \equiv \text{diag}(m_{1D}, m_{2D}, m_{3D}) \quad (6)$$

is the diagonalized Dirac mass matrix of neutrinos,  $m_{iD}$  are the eigenvalues of  $m_D$ . In (5) the  $M_R$  is the Majorana mass matrix in the basis where  $m_D$  is diagonal. This basis can be called the Dirac basis. Further on we will study properties of the  $M_R$  in this Dirac basis. For simplicity we will suggest that all matrices are real, although we will admit both negative and positive values of masses, which corresponds to different CP-parities of neutrinos.

The Eq. (5) can be considered as the relation between mass matrix of light neutrinos,  $m_{ss}$ , in the Dirac basis, the mass matrix of the RH neutrinos and the eigenvalues of the Dirac mass matrix. We will use this relation to study properties of the intermediate scale. Indeed,

(1) the eigenvalues of  $m_{ss}$  are the masses of light neutrinos which will be taken from experiment.

(2) According to the quark-lepton symmetry the eigenvalues  $m_{iD}$  can be related to masses of upper quarks

$$m_{iD} = k_i m_i^{\text{up}} \quad (7)$$

at some unification scale (e.g. at GU) and  $k_i$  are coefficients of the order 1 - 3.

(3) The mass matrix  $m_{ss}$  gives an additional contribution to the lepton mixing:  $V_s \neq I$ . In two generation case  $V_s$  is parametrized by one angle  $\theta_s$  which we call the see-saw angle ([6]) and total lepton mixing is

$$\theta_l = \theta_D + \theta_s, \quad (8)$$

where  $\theta_D$  follows from the Dirac mass matrix and can be related to the quark mixing angle. The data on lepton mixing then restrict  $\theta_s$ , and consequently  $M_R$ .

### 3 Mass Relation

Majorana mass matrix  $M_R$  can be written as

$$M_R = S_R M_R^{diag} S_R^T . \quad (9)$$

where

$$M_R^{diag} \equiv diag(M_1, M_2, M_3), \quad (10)$$

and  $M_i$  ( $i = 1, 2, 3$ ) are the masses of the RH neutrinos. For fixed values of  $m_i$  and  $m_{iD}$  the masses  $M_i$  depend on mixing, i.e. on  $S_R$ . However, there is a relation between masses which does not depend on the mixing. Calculating the determinants in the LH side and in the RH side of the see-saw formula (1) one finds

$$M_1 \cdot M_2 \cdot M_3 = \frac{m_{1D}^2 \cdot m_{2D}^2 \cdot m_{3D}^2}{m_1 \cdot m_2 \cdot m_3} , \quad (11)$$

where the RH side can be in principle determined from the experiment. For example,  $m_2$  can be fixed by solar neutrino data,  $m_3$  can be restricted by cosmological data, or by atmospheric neutrino data or/and by accelerator experiments. However, it will be difficult to get the information on  $m_1$  in the case of strong mass hierarchy. Moreover, for  $m_1 < 10^{-5}$  eV the see - saw contribution may be smaller, than e.g. gravitationally induced mass. In this case even known value  $m_1$  is useless for the determination of the intermediate scale masses. In this connection we will consider the task for the second and the third generations and then estimate possible influence of the first generation. The expression (11) can be rewritten as

$$M_2 \cdot M_3 = \xi_1 \frac{m_{2D}^2 \cdot m_{3D}^2}{m_2 \cdot m_3} , \quad (12)$$

where

$$\xi_1 \equiv \frac{m_{1D}^2}{M_1 m_1} . \quad (13)$$

If  $\xi_1 = 1$  the task is reduced to two neutrino task. The influence of the first generation is strong if  $\xi_1$  strongly deviates from 1 (see sect. 7).

## 4 Masses at zero mixing. Averaged mass scale and mass hierarchy parameter

We will consider first the case of two generations suggesting that  $\xi_1$  is close to 1. Let us introduce two mass parameters

$$M_{02} \equiv \frac{m_{2D}^2}{m_2}, \quad M_{03} \equiv \frac{m_{3D}^2}{m_3}, \quad (14)$$

where all the masses are taken at the electroweak scale. Evidently they coincide with masses of the RH neutrinos in absence of mixing in  $M_R$  (in Dirac basis, where  $m_D$  is diagonal).

Masses  $M_{02}$  and  $M_{03}$  can be determined from low energy data as follows. Suppose that the solar neutrino problem is solved by small mixing MSW solution, then in the case of mass hierarchy one has (see e.g. [8])

$$m_2 = \left(2.5 \pm_{0.7}^{0.9}\right) \cdot 10^{-3} \text{ eV}. \quad (15)$$

Suppose that at the Gran Unification scale,  $M_{GU} = 2 \cdot 10^{16} \text{ GeV}$ :

$$m_{2D} = k_2 m_c, \quad (16)$$

where  $m_c$  is the mass of charm quark. Then from (14) one gets  $M_{02}(m_Z)$  at  $m_Z$  :

$$M_{02}(m_Z) = \eta_2^2 k_2^2 \frac{m_c^2(m_Z)}{m_2(m_Z)}, \quad (17)$$

where  $\eta_2$  is the renormalization group factor corresponding to the boundary condition (16).

At one loop level

$$\eta_2 \approx \frac{E_\nu}{E_c}, \quad (18)$$

where  $E_\nu$  and  $E_c$  describe renormalization effect respectively for the Dirac neutrino mass and charm mass between  $m_Z$  and  $M_{GU}$ . The effect of large Yukawa couplings from third generation is the same for  $m_{2D}$  and  $m_c$ , so that only gauge interactions are important. Using the MSSM particle content and values  $m_c(m_Z) \approx 0.67 \text{ GeV}$ ,  $m_2(m_Z) \approx m_2(0)$  we get  $\eta_2 \approx 0.44$  and

$$M_{02} = (3.5 \pm 1.3) \cdot 10^{10} k_2^2 \text{ GeV}. \quad (19)$$

Where the error is rough estimation of uncertainties in the input parameters ( $\alpha_3$ , etc ). Since  $M_{02}$  does not run up to the intermediate scale the above value gives  $M_{02}$  at  $M_{02}$ .

Similarly,  $M_{03}$  can be related to the top quark mass:

$$M_{03} = \eta_3^2 \frac{m_t^2(m_Z)}{m_3(m_Z)}. \quad (20)$$

For third generation of fermions we take  $k_3 = 1$ , as could be hinted by  $b - \tau$  mass unification. The renormalization group factor  $\eta_3$  is

$$\eta_3 \approx \frac{E_\nu}{E_c} \frac{D_\tau}{D_b} \left( \frac{D_\nu}{D_t} \right)^3, \quad (21)$$

where  $D_i$  describe the Yukawa coupling renormalization effect. In particular,  $D_\nu$  is the renormalization effect of the neutrino Yukawa coupling between  $M_{03}$  and  $M_{GU}$ . (For this estimation we took  $M_{03} \sim 10^{12}$  GeV). Numerically [12]:

$$M_{03} = (2.1 \pm 0.7) 10^{12} \left( \frac{5 \text{ eV}}{m_3} \right) \text{ GeV}. \quad (22)$$

The cosmological bound on neutrino mass is  $m_3 < 50$  eV (for value of Hubble constant  $h \sim 0.7$ ) [9] gives according to (22)  $M_{03} > 2 \cdot 10^{11}$  GeV. More strong bound can be obtained from the Large scale structure of the Universe [10]. Pure hot dark matter scenario contradicts the observed structure. The hot dark matter contribution to energy density should be at least two times smaller than the contribution of the cold dark matter: i.e.  $\Omega_\nu < 0.3$ . This gives a conservative bound on the mass [10]:

$$m_3 < 15 \text{ eV} \quad (23)$$

which in turn leads according to (22) to the bound on the Majorana mass

$$M_{03} > 6 \cdot 10^{11} \text{ GeV}. \quad (24)$$

The best fit of Large scale structure of the Universe corresponds to  $m_3 \sim 5$  eV [11] which results in

$$M_{03} = (2.1 \pm 0.7) \cdot 10^{12} \text{ GeV}. \quad (25)$$

For  $m_3 \sim 0.1$  eV needed to solve the atmospheric neutrino problem in terms of neutrino oscillations one gets

$$M_{03} = (1.2 \pm 0.3) \cdot 10^{14} \text{ GeV}. \quad (26)$$

In what follows we will consider properties of the intermediate scale for two scenarios of masses and mixing of light neutrinos. (1) “Solar + HDM” neutrino scenario. It incorporates the MSW solution of solar neutrino problem and supplies the HDM component in the Universe. (2) “Solar + atmospheric” neutrino scenario. This scenario solves simultaneously the solar and the atmospheric neutrino problems. Let us stress that forthcoming experiments will allow to distinguish these two cases.

The immediate observation is that at least for  $k = 1$

$$M_{02} \ll M_{03}, \quad (27)$$

i.e. there is no unique scale for the RH neutrinos even for “solar + HDM” scenario.

The masses  $M_{02}$  and  $M_{03}$  allow one to introduce the average mass scale  $M_0$ :

$$M_0 \equiv \sqrt{M_{02}M_{03}}, \quad (28)$$

and the mass hierarchy parameter

$$\epsilon_0 \equiv \frac{M_{02}}{M_{03}}. \quad (29)$$

As we will see these two parameters determine important characteristics of the intermediate scale: in particular they give the natural ranges of masses and mixing in the RH sector.

From (19) and (25) one gets for scenario “solar + HDM” at  $k = 1$ :

$$M_0 = 2.8 \cdot 10^{11} \text{ GeV}, \quad \epsilon_0 = 1.3 \cdot 10^{-2}. \quad (30)$$

The bound from the Large scale structure of the Universe results in

$$M_0 > 1.3 \cdot 10^{11} \text{ GeV}, \quad \epsilon_0 < 10^{-1}. \quad (31)$$

In the case of “solar + atmospheric” neutrino scenario one has according to (19) and (26)

$$M_0 = 2.0 \cdot 10^{12} \text{ GeV}, \quad \epsilon_0 = 2.7 \cdot 10^{-4}. \quad (32)$$

Due to the relation (11) the average mass scale determines the product of the masses of the RH neutrinos for arbitrary mixing in the RH sector:

$$M_2 \cdot M_3 = M_0^2 \quad (33)$$



## 5 Universal scale

Let us consider first the possibility to have the universal mass scale for all RH neutrinos

$$M_1 = M_2 = M_3 \sim 2 \cdot 10^{12} \text{ GeV}, \quad (34)$$

where the number is taken from (25). In this case there is no mixing in the RH sector,  $V_s = I$ , and correspondingly,  $M_i = M_{0i}$ , lepton mixing is determined by Dirac mass matrices. To get  $M_{02} = M_{03}$  in the “solar + HDM” neutrino scenario one needs according to (19)  $k \approx 9$ . The neutrino mass,  $m_{2D}$ , can be enhanced in comparison with quark mass,  $m_c$ , e.g. if the element  $m_{23}^D$  of the neutrino Dirac mass matrix equals  $m_{23}^D \sim 3m_{23}^{up}$ . This can be achieved by the contribution of 126-plet in the  $SO(10)$  context.

Such a possibility faces however two problems:

(1). Mixing between  $\nu_\mu$  and  $\nu_\tau$  turns out to be strongly enhanced. The contribution from the neutrino mass matrix is

$$\theta_{\mu\tau}^\nu \sim 3\sqrt{\frac{m_c}{m_t}} \quad (35)$$

which gives  $\sin^2 2\theta_{\mu\tau} \approx 0.14$ . For  $m_3 \sim 5$  eV (in the cosmologically interesting region) this value is already excluded by the E531 experiment ( $\sin^2 2\theta_{\mu\tau} < 5 \cdot 10^{-3}$ ). CHORUS and NOMAD will further strengthen the bound. To satisfy the bound one should suggest strong cancellation of the contributions from neutrino and charge lepton sectors.

(2). On the contrary, neutrino contribution to mixing between the first and the second generations is suppressed. Even if the element  $m_{12}^D$  contains an additional factor 3 in comparison with corresponding quark mass, the contribution to mixing is

$$\theta_{e\mu}^\nu \sim \frac{1}{3}\sqrt{\frac{m_u}{m_c}}. \quad (36)$$

The total mixing angle,

$$\theta_{e\mu} > \sqrt{\frac{m_e}{m_\mu}} - \frac{1}{3}\sqrt{\frac{m_u}{m_c}} \quad (37)$$

is too big:  $\sin^2 2\theta_{e\mu} \sim 1.3 \cdot 10^{-2}$  is on the border of region of small mixing MSW solution to the solar neutrino problem.

Thus the possibility of the universal intermediate scale for “solar + HDM” scenario, although is not excluded, turns out to be strongly restricted by present data and will be checked by forthcoming experiments. Universal scale is practically excluded in the case “solar + atmospheric” neutrino scenario.

## 6 Mass hierarchy and mixing at the intermediate scale

In the case of two generations the mixing,  $S_R$ , in the RH sector is characterized by one angle  $\theta_M$ . According to (33) the product of masses  $M_2 \cdot M_3$  does not depend on  $\theta_M$ . However the masses and the mass ratio (mass hierarchy)

$$\epsilon \equiv \frac{M_2}{M_3} \quad (38)$$

do depend on mixing. Using the definition (38) and mass relation (33) we can write

$$M_2 = M_0 \sqrt{\epsilon}, \quad M_3 = M_0 \frac{1}{\sqrt{\epsilon}}. \quad (39)$$

Mixing in the RH sector changes the splitting between masses but it does not change the product of masses (for fixed masses of light neutrinos and Dirac mass terms).

Substituting (9,33) into see-saw formula (1) for two generations one finds relation between  $\theta_M$  and  $\epsilon$

$$\sin^2 \theta_M = \frac{1}{(1 - \epsilon)(1 - \epsilon_D^2)} \left[ \pm \left( 1 + \frac{m_2}{m_3} \right) \sqrt{\epsilon_0 \epsilon} - \epsilon - \epsilon_D^2 \right], \quad (40)$$

where

$$\epsilon_D \equiv \frac{m_{2D}}{m_{3D}} \quad (41)$$

is the mass hierarchy in Dirac sector. At  $m_2/m_3 \ll 1$  and  $\epsilon \gg \epsilon_D^2$  the expression (40) reduces to

$$\sin^2 \theta_M \approx \frac{\epsilon}{1 - \epsilon} \left[ \pm \sqrt{\frac{\epsilon_0}{\epsilon}} - 1 \right]. \quad (42)$$

The latter does not depend on  $m_2/m_3$  and  $\epsilon_D^2$  explicitly. These values enter only via  $\epsilon_0 = \epsilon_D^2/(m_2/m_3)$ .

Diagonalization of the see-saw mass (5) matrix gives for the see-saw angle

$$\tan 2\theta_s = \frac{\sin 2\theta_M \epsilon_D (1 - \epsilon)}{\epsilon - \epsilon_D^2 + \sin^2 \theta_M (1 - \epsilon)(1 + \epsilon_D^2)} \quad (43)$$

Substituting  $\sin^2 \theta_M$  from (42) in this expression we get for  $\epsilon \gg \epsilon_D^2$ :

$$\sin^2 \theta_s \approx \frac{\epsilon_D^2}{\epsilon_0} \left[ \pm \sqrt{\frac{\epsilon_0}{\epsilon}} - 1 \right]. \quad (44)$$

The Eq. (40,43) or (42,44) are the basic relations we will use to study the properties of the intermediate scale. The effect of mixing in the intermediate scale is different for  $\epsilon > 0$  and  $\epsilon < 0$ , i.e. for the cases of equal and opposite CP - parities of neutrinos. We will consider these two cases separately.

## 6.1 $\epsilon_0 > 0$

If  $\epsilon_0$  is positive then according to (42)  $\epsilon$  should be also positive which means that neutrinos have the same CP - parity. From positivity of the RH side of Eq. (42) it follows that only sign plus is possible, and moreover

$$\epsilon > \epsilon_0, \quad (45)$$

i.e. mixing at the intermediate scale always enhances the hierarchy of masses (for fixed values of  $m_i$  and  $m_{iD}$ ).

According to (42) mixing angle  $\theta_M$  first increases with  $\epsilon$ , then reaches maximum value

$$\sin^2 2\theta_M^{max} \approx \epsilon_0 \quad \text{at} \quad \epsilon \approx \frac{\epsilon_0}{4}, \quad (46)$$

and then decreases again. It becomes zero at

$$\epsilon \approx \frac{\epsilon_D^4}{\epsilon_0}. \quad (47)$$

For very strong mass hierarchy,  $\epsilon \ll \epsilon_D^2$ , the expression for mixing angle can be approximated as

$$\sin^2 \theta_M \approx \sqrt{\epsilon_0 \epsilon} - \epsilon - \epsilon_D^2. \quad (48)$$

Thus there is maximal value of mixing,  $\theta_M \approx \sqrt{\epsilon_0}/2$ , in the RH sector determined by parameter  $\epsilon_0$ . The hierarchy parameter is restricted by  $\epsilon_D^4/\epsilon_0 \leq \epsilon \leq \epsilon_0$ .

The values of angle and mass hierarchy

$$\theta_M \sim \frac{\sqrt{\epsilon_0}}{2}, \quad \epsilon \sim \left(\frac{1}{4} - \frac{1}{2}\right) \epsilon_0 \quad (49)$$

can be considered as the natural values. They do not imply any fine tuning of parameters and any additional symmetry in  $M_R$ .

In the limit of very strong hierarchy,  $\epsilon \ll \epsilon_0$  one has from (48)

$$\sin^2 \theta_M \approx \sqrt{\epsilon_0 \epsilon} \gg \epsilon. \quad (50)$$

This inequality implies smallness of the determinant of  $M_2$ :

$$\frac{Det M_R}{M_{23}^2} \approx \frac{\epsilon}{\sin^2 \theta_M} \sim \sqrt{\frac{\epsilon}{\epsilon_0}} \ll 1, \quad (51)$$

i.e. near to singular character of  $M_R$ . This in turn means fine tuning of elements of the matrix in the Dirac basis.

Let us consider now the dependence of the see-saw angle on the mass hierarchy. According to (44) the angle is zero at  $\epsilon = \epsilon_0$ , it increases with diminishing  $\epsilon_M$ . In the range  $4\epsilon_D^4/\epsilon_0 \ll \epsilon \ll \epsilon_0$  the dependence can be approximated by (see (44)):

$$\sin^2 \theta_s \approx \frac{4\epsilon_D^2}{\sqrt{\epsilon_0 \epsilon}} \quad (52)$$

(this formula is true when  $\sin^2 2\theta_s$  is still much smaller than 1). With further decrease of  $\epsilon$  the mixing approaches maximal value ( $\sin^2 2\theta_s = 1$ ) at

$$\epsilon = \frac{4\epsilon_D^4}{\epsilon_0}, \quad (53)$$

and then decreases up to zero at  $\epsilon = \frac{\epsilon_D^4}{\epsilon_0}$  (see (48)). Very strong hierarchy leads to strong see-saw enhancement of lepton mixing [6].

Let us consider the applications of the results. The natural value of mass hierarchy in the case “solar + HDM” neutrino scenario is  $\epsilon \sim \epsilon_0/4 \sim (3-4) \cdot 10^{-3}$ . It coincides with hierarchy in Dirac sector:  $\epsilon \approx \epsilon_D$ . Thus “solar + HDM” neutrino scenario implies linear hierarchy of masses of the RH neutrinos. This may be considered as a hint to the common origin of the Yukawa couplings which generate the Dirac and the Majorana mass matrices. Numerically, for  $\epsilon = \epsilon_0/3$  we get  $M_2 = 2 \cdot 10^{10}$  GeV and  $M_3 = 5 \cdot 10^{12}$  GeV. These masses could be generated by the interaction with scalar field acquiring the VEV  $V \sim 10^{13}$  GeV.

Lower bound on mass hierarchy follows from experimental bound on mixing between the second and the third generations. Indeed, if there is no strong cancellation between the Dirac matrix and see-saw matrix contributions, so that

$$\sin^2 2\theta_s < \sin^2 2\theta_{\mu\tau}, \quad (54)$$

then one gets according to (44)

$$\epsilon > \frac{\epsilon_0}{\left(1 + \sin^2 2\theta_{\mu\tau} \cdot \frac{\epsilon_0}{4\epsilon_D^2}\right)^2} \quad (55)$$

and present experimental bound:  $\sin^2 2\theta_{\mu\tau}$  at  $\Delta m^2 > 25 \text{ eV}^2$  gives

$$\epsilon > \frac{\epsilon_0}{9}. \quad (56)$$

For  $k = 1$  this leads to bounds on the RH masses  $M_2 > 10^{10} \text{ GeV}$  and  $M_3 < 10^{13} \text{ GeV}$ . CHORUS and NOMAD will further strengthen the bound on hierarchy up to  $\epsilon > (1/4 - 1/2)\epsilon_0$  and therefore squeeze the intervals for masses.

In principle, the mass  $m_3$  in the cosmologically interesting region does not necessarily imply  $M_{33} < 10^{13} \text{ GeV}$ . Admitting the tuning of the  $DetM_R$  (51) and strong cancellation between the Dirac and the see-saw contributions to the lepton mixing (to satisfy the experimental bounds) one can enhance the hierarchy and therefore push  $M_3$  to higher values. The increase of  $M_3$  diminishes (removes) the renormalization effect due to large Yukawa coupling of neutrino from the third generation [12, 13]. According to (39) the mass  $M_3 \sim 10^{16} \text{ GeV}$  can be achieved for the hierarchy parameter  $\epsilon = (M_0/M_{GU})^2 \sim 10^{-8}$ . In this case as follows from (51) the level of fine tuning is  $10^{-3}$ .

On the contrary, in “solar + atmospheric” scenario one can use the limit of strong mass hierarchy to enhance the mixing and thus to explain the atmospheric neutrino deficit. According to (53) for  $\epsilon_D = 3 \cdot 10^{-3}$  and  $\epsilon_0 = 2.7 \cdot 10^{-4}$  maximal see-saw mixing is achieved at  $\epsilon \sim 10^{-6}$ . This corresponds to the mass of third neutrino  $M_3 \approx 2 \cdot 10^{15} \text{ GeV}$ , i.e. of the order of  $M_{GU}$ . Second neutrino becomes rather light:  $M_2 \sim 2 \cdot 10^9 \text{ GeV}$ . There is no usual intermediate scale in this case. In this connection let us mark the following possibility. At the Grand Unification scale only  $M_3$  acquires mass, whereas  $M_2$  and  $M_1$  are massless. Masses of the second and first neutrinos appear as the result of violation of certain horizontal symmetry, by e.g. nonrenormalizable interactions, so that  $M_2 \sim M_{GU}(M_{GU}/M_{Pl})^2$ .

## 6.2 $\epsilon < 0$

According to (42)  $\epsilon_0$  should be also negative. However, as follows from (42) the mass hierarchy can be both stronger (sign plus in front of square root) and weaker (sign plus and minus) than  $\epsilon_0$ . Let us consider the case of weaker hierarchy:

$$|\epsilon| > |\epsilon_0| \quad (57)$$

There is no upper bound on mixing angle  $\theta_M$ . With increase of  $\epsilon$  the  $\sin^2 \theta_M$  increases; it equals

$$\sin^2 \theta_M = \frac{1}{2}(1 \pm \sqrt{|\epsilon_0|}) \approx \frac{1}{2} \quad (58)$$

at  $|\epsilon| = 1$ , i.e. for equal absolute values of the RH neutrino masses. Thus to have  $|M_2| \approx |M_3|$  one needs practically maximal mixing in the RH mass matrix. Note that in all region  $\sin^2 \theta_M \sim \epsilon$ , i.e. naturalness criteria is fulfilled.

The see-saw angle is determined by

$$\sin^2 \theta_s \approx \frac{\epsilon_D^2}{\epsilon_0} \left[ 1 - |\epsilon_0| - \sqrt{\frac{\epsilon_0}{\epsilon}}(1 - |\epsilon|) \right] \approx \frac{\epsilon_D^2}{\epsilon_0} \approx \frac{m_2}{m_3} \quad (59)$$

for  $|\epsilon| \sim 1$ . Using the value  $\epsilon_0$  (30) we find  $\sin^2 2\theta_s \approx 3 \cdot 10^{-3}$  which is near the existing experimental bound for large mass splitting (“solar + HDM”). In the case  $|\epsilon| \sim 1$  the mass matrix  $M_R$  is strongly off diagonal:  $M_{22} = -M_{33} \propto \sqrt{\epsilon_0}$  and  $M_{23} \propto \sqrt{1 - \epsilon_0}$ .

Mixing allows one to relax mass hierarchy and therefore diminish the value of  $M_3$ . In this connection let us mark two examples in the “solar + atmospheric” neutrino scenario.

(1). Linear mass hierarchy:  $\epsilon \sim 3 \cdot 10^{-3}$ . In this case for  $k = 1$  we get  $M_3 = 3 \cdot 10^{13}$  GeV,  $M_2 = 1.2 \cdot 10^{11}$  GeV,  $\sin^2 \theta_M = 2 \cdot 10^{-3}$ , and  $\sin^2 2\theta_s = 2 \cdot 10^{-3}$ .

(2). Equal masses:  $|M_2| \sim |M_3| \sim M_0$ . In this case diagonal elements of matrix  $M_R$  are suppressed in comparison with off diagonal by  $\sqrt{\epsilon_0} \approx 10^{-2}$ . The largest element of the mass matrix  $M_{23} \approx M_0 \approx 2 \cdot 10^{12}$  GeV. If  $m_2 = 4 \cdot 10^{-3}$  eV and  $m_3 = 3 \cdot 10^{-2}$  eV then according to (59)  $\sin^2 2\theta_s \sim 0.5$  in the region of solution of the atmospheric neutrino problem.

Thus it is possible to explain simultaneously both the solar and the atmospheric neutrino problems by pseudo Dirac structure at the intermediate scale  $M = (2 - 3) \cdot 10^{12}$  GeV. Such a structure can be obtained by imposing, e.g.,  $U(1)_G$  horizontal symmetry with charge prescription  $(0, -1, +1)$ , then scalar with zero G- charge will produce the mass matrix

$$M_R = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & 0 & M \\ 0 & M & 0 \end{pmatrix}. \quad (60)$$

The violation of this symmetry should be characterized by factor  $10^{-2}$ .

## 7 Effect of first generation

If mixing of the first generation with second and third is sufficiently small, then the task is reduced to two neutrino task and the results for three generations coincide with those obtained in the previous sections. Smallness of mixing needed to solve the solar neutrino problem by the MSW effect may indicate on such a weak influence. Let us consider situations when influence of the first generation is strong.

As we marked in sect.3 the influence of first generation can be characterized by parameter  $\xi_1$  in such a way that strong deviation of  $\xi_1$  from 1 means the strong effect of the first generation.

Simple dependence of  $\xi_1$  on the matrix element  $M_{12}$  can be found explicitly for the case when first generation mixes with second generation only.

Parameter  $\xi_1$  as function of  $M_{12}$  is different for two cases depending on whether  $M_{22}$  is larger or smaller than  $M_{11}/\epsilon_D'^2$ , where  $\epsilon_D'^2 \equiv m_u/m_c$ .

(a).  $M_{22} < M_{11}/\epsilon_D'^2$ . Main features of the dependence  $\xi_1(M_{12})$  are the following.

- (i)  $\xi_1 = 1$  at  $M_{12} = 0$  ( the first family decouples).
- (ii)  $\xi_1$  increases with  $M_{12}$  and its dependence can be approximated by

$$\xi_1 \sim \frac{M_{11}M_{22}}{D_{12}}, \quad (61)$$

where  $D_{12} \equiv \text{Det}M \equiv M_{11}M_{22} - M_{12}^2$  in the region

$$M_{12} \sim \sqrt{M_{11}M_{22}}. \quad (62)$$

According to (61)  $\xi_1 \rightarrow \infty$ , when  $D_{12} \rightarrow 0$ .

(iii) At  $M_{12} > \sqrt{M_{11}M_{22}}$ , the parameter  $\xi_1$  changes the sign and its absolute value decreases with further increase of  $M_{12}$ . For example at  $M_{12} \sim M_{22}/2$ :

$$\xi_1 = \frac{\epsilon_D'^2 M_{22}}{M_{11}} \frac{1}{(\sqrt{2} - 1)^2} \quad (63)$$

and if  $M_{11}/M_{22} \sim \epsilon_D$  one gets  $\xi_1 \ll 1$ .

(iv) For  $M_{12} \gg M_{22}$ , i.e. when nondiagonal element dominates one gets:  $\xi_1 \rightarrow \epsilon_D$ .

(b).  $M_{22} > \frac{M_{11}}{\epsilon_D'^2}$ . (This case covers in particular the mass matrices satisfying Fritzsch ansatz ( $M_{11} = 0$ )). Now  $\xi_1$  monotonously decreases from  $\xi_1 = 1$  at  $M_{12} = 0$  to  $\xi_1 \approx \epsilon_D$  for  $M_{12} \gg M_{22}$ .

In particular, for  $M_{12} \sim \sqrt{M_{11}M_{22}}$  the  $\xi_1$  dependence can be approximated by

$$\xi_1 \approx \frac{M_{22} + \frac{1}{\epsilon_D^2} M_{11}}{M_{22} + M_{11}} \quad (64)$$

and evidently  $\xi_1 < 1$  in contrast with the case (a).

Thus  $\xi_1$  strongly deviates from 1 and there is a strong influence of the first generation in two cases:

(1) If  $D_{12} \rightarrow 0$  then  $\xi_1 \rightarrow \infty$ .

(2) If  $M_{12} > M_{22}$  then  $\xi_1 \ll 1$ .

The bound on the mixing of light neutrinos restricts the effect of the first generation. Diagonalizing  $m_s$  explicitly, we get the see-saw mixing between first and second generations

$$\tan 2\theta_s^{12} = \frac{2\epsilon_D' M_{12}}{M_{11} - \epsilon_D'^2 M_{22}}. \quad (65)$$

Solution of the solar neutrino problem implies  $m_1 < m_2$  and therefore  $\epsilon_D'^2 M_{22} < M_{11}$ . That is only the case (a) gives the solution of the problem. For small mixing solution the angle  $\theta_s^{12}$  can be restricted, if the Dirac mixing is similar to that in quark sector. In this case one has  $\tan 2\theta_s^{12} < \tan 2\theta_\odot \approx 0.1$  (the value needed to explain the solar neutrino problem) and then from (65)

$$\frac{2\epsilon_D' M_{12}}{M_{11}} < \tan 2\theta_\odot \sim 0.1. \quad (66)$$

Let us consider the applications of the results to our analysis in sect. 2 - 6.

(1) If  $D_{12} \rightarrow 0$ , and therefore  $\xi_1 \rightarrow \infty$ , the average scale increases:

$$M_2 \cdot M_3 = \xi_1 M_0^2. \quad (67)$$

Substituting  $M_{12} \sim \sqrt{M_{11}M_{22}}$  into (66) we find

$$\frac{M_{11}}{M_{22}} > \frac{4\epsilon_D'^2}{\tan^2 2\theta_\odot} \sim 5 \cdot 10^{-3}, \quad (68)$$

i.e.  $M_{11} \gg \epsilon_D'^2 M_{22}$  which gives  $m_2 \approx m_c^2/M_{11}$ . From this relation we have

$$M_{22} = M_{02} \left( \frac{M_{11} M_{22}}{D_{12}} \right). \quad (69)$$



Thus diminishing  $D_{12}$  one can push the value of  $M_{22}$  and therefore  $M_2$  up. In this case also  $M_{11}$  should increase according to (68). The mass  $m_1 \approx m_u^2/M_{11}$  turns out to be small. Numerically one may have  $M_{22} \sim M_{33} \sim 2 \cdot 10^{12} \text{ GeV}$ ,  $M_{11} > 10^{10} \text{ GeV}$  and  $M_{12} > 1.4 \cdot 10^{11} \text{ GeV}$ .

(2) If  $M_{12} > M_{22}$  then  $\xi_1 \ll 1$  which leads to decrease of  $M_2 \cdot M_3$ . Bound on the see-saw mixing angle results in restriction  $M_{12}/M_{11} < \tan 2\theta_\odot/(2\epsilon_D) \sim 16$  and this leads to bound on  $\xi_1$ .

## 8 Conclusion

1. Simple relations have been derived between parameters of the Intermediate scale and neutrino data in context of the see-saw mechanism of the neutrino mass generation and quark - lepton symmetry.

2. Neutrino masses hinted by the solar neutrino data, the Large Scale Structure of the Universe or atmospheric neutrino anomaly allow to introduce mass parameters  $M_{02}$  and  $M_{03}$  which in turn determine the average mass scale  $M_0$  and the mass hierarchy  $\epsilon_0$ .

The scale  $M_0$  and the mass hierarchy  $\epsilon_0$  fix natural ranges of masses  $M_2, M_3$  and mixing of the RH neutrinos. Strong deviations from these natural ranges imply certain symmetry or/and fine tuning of elements of  $M_R$ . Namely, the mass matrix should be strongly off diagonal, which may be stipulated by certain horizontal symmetry, or its determinant should be much smaller than nondiagonal element squared (in the basis where neutrino Dirac mass matrix is diagonal). This implies certain correlation between Yukawa coupling generating the Dirac and the Majorana mass matrices.

3. Using the solar, atmospheric, and cosmological data one can make the following tentative conclusions:

(a). In the case of “solar + HDM” scenario with  $\epsilon > 0$  the masses of the RH neutrinos are restricted by the following intervals:  $M_2 = (1 - 4) \cdot 10^{10} \text{ GeV}$  and  $M_3 = (2 - 8) \cdot 10^{12} \text{ GeV}$ . The mass hierarchy is bounded by  $\epsilon = (1/9 - 1)\epsilon_0$ , where the upper edge corresponds to the absence of mixing in the RH sector, and the lower edge follows from the experimental bound on mixing of light neutrinos. Thus one gets the linear hierarchy of RH neutrinos

masses:  $M_i \propto m_{iD} \sim m_{iup}$  with largest mass  $M_3 \approx 5 \cdot 10^{12}$  GeV. This may testify for (1) simple relation (equality) of the Yukawa couplings which generate masses of up quarks and Majorana masses of RH neutrinos, for (2) spontaneous violation of symmetry of lepton number at  $V \sim 10^{13}$  GeV.

Unique intermediate scale is not excluded for this scenario, but it implies rather strong deviation from simple relations between quark and lepton masses and it is on the border of already existing bounds.

(b). “Solar + atmospheric” neutrino scenario implies in general rather strong mass hierarchy in the RH sector. However if  $M_R$  has essentially off diagonal structure, all the masses of the intermediate scale below  $10^{13}$  GeV are not excluded.

Another possibility is that the largest mass  $M_3$  is of the order of  $M_{GU}$ . Then second mass should be below  $3 \cdot 10^9$  GeV. It can be generated by high order nonrenormalizable interactions. In this case there is strong see-saw enhancement of lepton mixing which allows one to solve the atmospheric neutrino problem.

4. First generation can strongly influence the average scale for second and third generation in two cases: (i) when  $2 \times 2$  matrix including first family is strongly off diagonal (ii) when the determinant of this matrix is much smaller than the off diagonal matrix element squared. In the former case the average scale decreases in the latter - increases. In the most natural case (taking into account small mixing solution of the solar neutrino problem) the influence of the first generation is not strong and results for two heavy generations can be changed by factor of the order 1.

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